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Adapted Non-Gaussian Mixture Model for Image Denoising

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Abstract—*In the recent three decades, number of methods are suggested for image denoising. Although most of the denoising methods give great results when implemented in real-time image denoising problems, but still there is a room for advancement. Therefore, we suggests a patch-based image denoising method with the non-gaussian mixture model (NGMM). In this method, image is partitioned into patches and each patch is denoised using NGMM. After denoising each patch, all patches are combined to form a complete denoised image. Moreover we also suggests an adaptive learning procedure for learning parameters of NGMM. Simulation results for proposed method are better than other existing image denoising methods.*

1. INTRODUCTION

Image denoising is an essential task in the field of image processing. Number of various image denoising methods are proposed in recent time. Before discussing various denoising methods, consider a classical ill-posed image denoising problem,

$$y = x + n$$

where original image x is corrupted by an additive independent and identical (i.i.d.) noise n. This results in noisy image y. Here aim is to obtain original image x from noisy image y. To achieve this aim various methods are suggested, such as: wavelet transform based methods [1], sparse coding based methods [2], non-local means (NLM) based methods [3], singular value decomposition (SVD) based methods [4], principal component analysis (PCA) based methods [5], expected log likelihood (EPLL) based method [6], fusion methods [7], and many more. [1]-[7], all are different denoising methods, however they have one common thing, i.e., they all are patch-based techniques.

In a patch-based technique, a complete noisy image is partitioned into various patches, and then each and every patch of image is denoised by taking help of some reference information. Reference information can be taken from noisy image itself or from some external database of images. Zontak *et. al*, in his paper [8] used internal statistics of image for denoising the image, whereas Chan *et. al,* in his paper [9] used statistics of external image database for denoising the image. If an external image database is built by using general random images then it is called as generic database, whereas if an external image database is built by using images which are relevant to the noisy image then that database is called as targeted database. Luo *et. al*, in [10] make use of targeted database. Furthermore researchers also used combination of internal and external database to denoise an image, refer to [11].

With patch-based method, modeling of image patch-priori is an important task. Hence, for modeling image patch-priori, gaussian distribution and its mixture model are popularly being used by many researchers [6,9]. Apart from image denoising, GMM is also being used in various other applications such as: image classification [12], and so on. But the drawback to gaussian distribution is that it is a thin-tailed distribution, to alleviate this drawback heavy-tailed distribution, e.g., Student t distribution, can be used as a robust solution to GMM. To support this argument we like to quote from Murphy's book [13], "*One problem with the Gaussian distribution is that it is sensitive to outliers, since the log probability decays quadratically with distance from center. A more robust alternative is Student t distribution.*" Thus Student t distribution and its mixture model (STMM), alike GMM, is also being used in various applications such as: image compression [14], image restoration [15], etc.

Henceforth in this paper, we used a patch-based framework for image denosing together with NGMM patch-priori. For estimating parameters of NGMM, an adaptive algorithm is proposed. The proposed algorithm is a modified version of popularly known algorithm, expectation-minimization (EM) algorithm. Our proposed work take the NGMM parameters which are learned from external reference, and adapts it to noisy image via pre-filtering stage.

The rest of the paper is organized as: After introduction, there are four more sections. In Section 2, a denoising procedure is presented with patch-priori modeled from NGMM. After that, in Section 3, an adaptive algorithm for estimating NGMM parameters is discussed. Furthermore, experimental results are given in Section 4, whereas in Section 5, concluding remark with future work is stated.

2. DENOISING BY PATCH PRIORI

In order to obtain original clean image x from noisy image y, we consider following optimization for image denoising problem,

$$\min_x [-\log f(y|x) - \log f(x)]$$

First term is calculated from noise model equation, and in this paper, we consider noise to be additive i.i.d. gaussian noise with zero mean. Thus, $\log f(y|x) = (2\sigma^2)^{-1}\ \|y-x\|^2$.

Second term is calculated using image priori. Since image x can be seen as a collection of patches, therefore image priori can be calculated by means of collection of patch-priori. Moreover, patch-priori are taken from STMM. Therefore, $\log f(x) = \sum_i \sum_j \log\left(w_j\, \mathcal{T}\left(P_i x \middle| \theta_j\right)\right)$, where, $\theta_j = \{\mu_j, \Sigma_j, \nu_j\}$, is the set of parameters of Student t distribution $\mathcal{T}\left(P_i x\middle|\theta_j\right)$, and w_j are mixing weights. The Student t distribution for d – dimensional x, with $\Gamma(\cdot)$ is gamma function, and θ is the set of parameters, correlation matrix Σ, mean vector μ, and degree-of-freedom (d.o.f.) ν, can be given as,

$$\mathcal{T}(x|\,\theta) = \frac{\Gamma\left(\frac{\nu+d}{2}\right)\ |\Sigma|^{-\frac{1}{2}}}{\Gamma\left(\frac{\nu}{2}\right)\ (\nu\pi)^{\frac{d}{2}}}\left[1+\frac{(x-\mu)^T\Sigma^{-1}(x-\mu)}{\nu}\right]^{-\frac{\nu+d}{2}}$$

Therefore, optimization problem becomes,

$$\min_x\left[-(2\sigma^2)^{-1}\|y-x\|^2 - \sum_i\sum_j \log\left(w_j\,\mathcal{T}\left(P_i x\middle|\theta_j\right)\right)\right]$$

By using half quadratic splitting method [6], objective function is augmented as,

$$\min_x\left[-(2\sigma^2)^{-1}\|y-x\|^2 + \frac{\beta}{2}\sum_i\|P_i x - u_i\|^2 - \sum_i\sum_j \log\left(w_j\,\mathcal{T}\left(P_i x\middle|\theta_j\right)\right)\right]$$

where β is called penalty parameter, and u_i are the set of auxiliary patches.

The solution to above problem can be estimated by alternating-minimization algorithm, and by assuming one dominating mixture component $k_i^* = \max_j w_j\,\mathcal{T}\left(P_i x\middle|\theta_j\right)$, [6,9,16]. Therefore,

$$x^{k+1} = \left((\sigma^2)^{-1} I + \beta\sum_i P_i^T P_i\right)^{-1}$$

$$\left((\sigma^2)^{-1} + \beta\sum_i P_i^T\, u_i^k\right)$$

$$u_i^{k+1} = u_i^k + \Delta t\left[(\sigma^2)^{-1}P_i(y-x) + \frac{\left(\nu_{k_i^*}+d\right)\left(P_i u^k - \mu_{k_i^*}\right)\left|\Sigma_{k_i^*}\right|^{-1}}{\nu_{k_i^*} + \delta_{k_i^*}}\right]$$

where $\delta_{k_i^*} = \left(P_i u^k - \mu_{k_i^*}\right)^T\left|\Sigma_{k_i^*}\right|^{-1}\left(P_i u^k - \mu_{k_i^*}\right)$.

3. ADAPTIVE ALGORITHM WITH NGMM

The EM algorithm is a widely used algorithm for estimating parameters of mixture model and it also guarantees its convergence [17]. For this reason, we used EM algorithm as a foundation stone for our algorithm.

Consider an image, whose patches $\{\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n\}$ are modeled from STMM having the set of parameter $\widetilde{\Theta}_j = \{\widetilde{w}_j, \tilde{\mu}_j, \tilde{\Sigma}_j, \tilde{\nu}_j\}$. And, another STMM having the set of parameter $\Theta_j = \{w_j, \mu_j, \Sigma_j, \nu_j\}$, are learned from external references. Now we wish to estimate $\widetilde{\Theta}_j$ by using Θ_j. Then, expectation (E), and maximization (M) steps of the EM algorithm can be given as,

E-step. In this step, we compute complete data log-likelihood, instead of computing simple log-likelihood, and it is termed as an auxiliary function. For any mixture model auxiliary function is calculated by the help of term called as responsibility. For STMM, responsibility is given as,

$$\gamma_{ij} = \frac{w_j\,\mathcal{T}\left(\tilde{p}_i\middle|\theta_j\right)}{\sum_l w_l\,\mathcal{T}(\tilde{p}_i|\theta_l)}$$

M-step. This step is a parameter estimation step, where auxiliary function is maximized iteratively with hyper-prior. Thus parameters of NGMM are given as, refer to [9,14],

$$\widetilde{w}_j = \alpha_j\frac{n_j}{n} + \left(1-\alpha_j\right) w_j$$

$$\tilde{\mu}_j = \alpha_j \frac{1}{n_j} \sum_i \gamma_{ij} \tilde{p}_i + \left(1 - \alpha_j\right) \mu_j$$

$$\tilde{\Sigma}_j = \alpha_j \frac{1}{n_j} \sum_i \gamma_{ij} \left(\tilde{p}_i - \tilde{\mu}_j\right) \left(\tilde{p}_i - \tilde{\mu}_j\right)^T$$

$$+ \left(1 - \alpha_j\right) \left(\Sigma_j + \left(\mu_j - \tilde{\mu}_j\right)\left(\mu_j - \tilde{\mu}_j\right)^T\right)$$

where, $n_j = \sum_i \gamma_{ij}$, $\alpha_j = \frac{n_j}{n_j + \rho}$, and ρ is called as the factor of relevance. The solution to d.o.f. $\tilde{v}_j$ is obtain by solving following equation,

$$-\psi\left(\frac{\tilde{v}_j}{2}\right) + \log\frac{\tilde{v}_j}{2} + 1 + \frac{1}{n_j} \sum_i \gamma_{ij} \left(\log z_i - z_i\right) + \psi\left(\frac{\tilde{v}_j + d}{2}\right) - \log\left(\frac{\tilde{v}_j + d}{2}\right) = 0$$

where $\psi(\cdot)$ is digamma function. At this point, important to note that the procedure stated above is for noise-free patches, and practically there is no such image patches which are not affected by noise. This is true because various type of noise are added in various stages. So for this reason, we suggest pre-filtering of noisy image. Pre-filtering can be done by implementing any existing denoising algorithm. This is in turn help us to obtain more satisfying results for denoising issue.

4. EXPERIMENTAL RESULTS

In our experiment, we learn NGMM with 200 mixture components from an external generic database having image patches of size 8x8, and for testing, we used two standard, boat and pepper, images of size 256x256.

Labelling our proposed algorithm as ANGMM, and comparison method proposed for GMM [9] as AGMM. On comparing ANGMM with AGMM, we obtain better results, in term of peak-signal-to-noise ratio (PSNR). Table 1, summarizes the PSNR results for two standard images with σ = 20. In both cases of boat and pepper images, we obtained higher PSNR values for ANGMM. Thus, with obtained practical results we can say that STMM is superior to GMM for image denoising.

Figure 1 shows denoising results of standard, boat and pepper, images. In Figure 1, the top most layer of images are an original images, then noisy images are placed. After that, denoised images by AGMM and ANGMM are placed one after other. Therefore, our proposed adaptive algorithm is a better option for image denoising.

5. CONCLUSION AND FUTURE WORK

In this paper, first we discussed a patch-based image denoising method, and used NGMM in place of GMM for modeling patch-priori. The reason for replacing GMM with STMM (NGMM) is that, Student t distribution is heavy-tail distribution, and so its mixture model is a more robust solution than GMM. Second, the proposed adaptive algorithm, which is the key contribution of the paper, trains NGMM patch-priori. Furthermore, modification to proposed method by pre-filtering stage is also stated. In comparison with other existing image denoising algorithm, experimental results showed better performance of proposed algorithm in terms of PSNR.

Future work will be devoted to detailed analysis of proposed algorithm for extending this work to image deblurring, and image inpainting.

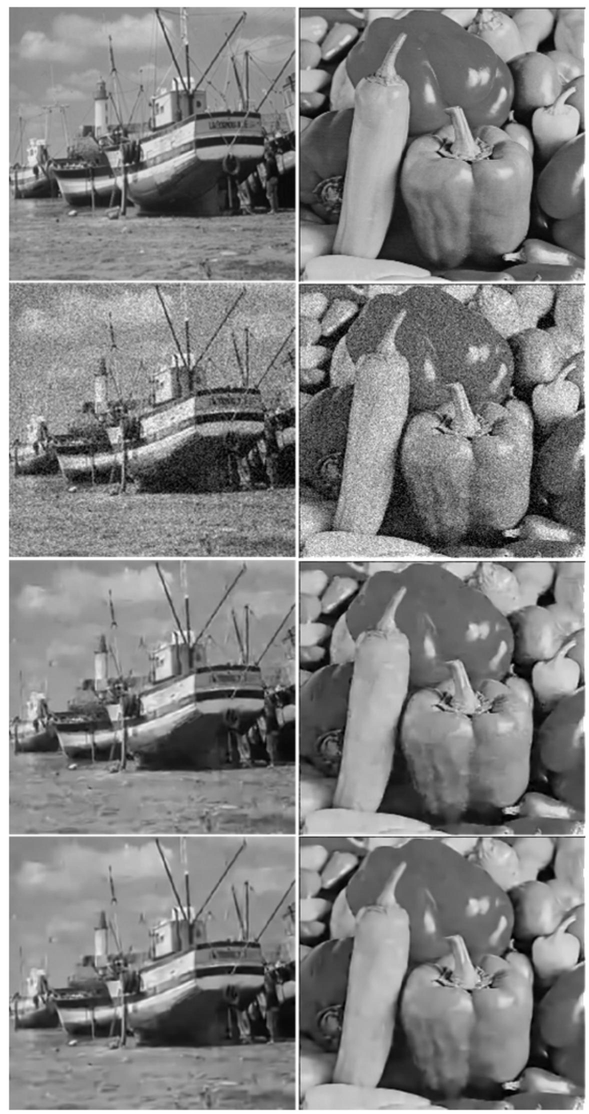

Fig. 1: Denoising result of standard 256x256 images.

Table 1: PSNR results

Standard Image	AGMM	ANGMM
Boat	28.72	28.91
Peppers	30.22	30.48

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